ADVANCED SUBSIDIARY GCE UNIT **4755/01**

MATHEMATICS (MEI)

Further Concepts for Advanced Mathematics (FP1)

THURSDAY 18 JANUARY 2007

Afternoon

Time: 1 hour 30 minutes

Additional materials:
- Answer booklet (8 pages)
- Graph paper
- MEI Examination Formulae and Tables (MF2)

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.

ADVICE TO CANDIDATES

- Read each question carefully and make sure you know what you have to do before starting your answer.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.

[Turn over

Section A (36 marks)

1 Is the following statement true or false? Justify your answer.

$$x^2 = 4 \text{ if and only if } x = 2 \qquad [2]$$

2 **(i)** Find the roots of the quadratic equation $z^2 - 4z + 7 = 0$, simplifying your answers as far as possible. [4]

(ii) Represent these roots on an Argand diagram. [2]

3 The points A, B and C in the triangle in Fig. 3 are mapped to the points A′, B′ and C′ respectively under the transformation represented by the matrix $\mathbf{M} = \begin{pmatrix} 2 & 0 \\ 0 & \frac{1}{2} \end{pmatrix}$.

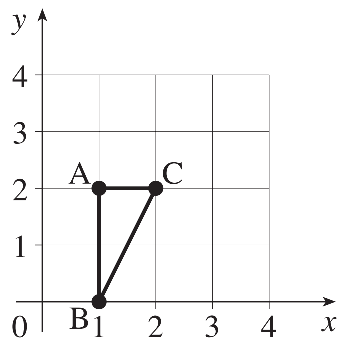

Fig. 3

(i) Draw a diagram showing the image of the triangle after the transformation, labelling the image of each point clearly. [4]

(ii) Describe fully the transformation represented by the matrix **M**. [3]

4 Use standard series formulae to find $\sum_{r=1}^{n} r(r^2 + 1)$, factorising your answer as far as possible. [6]

5 The roots of the cubic equation $2x^3 - 3x^2 + x - 4 = 0$ are α, β and γ.

Find the cubic equation whose roots are $2\alpha + 1$, $2\beta + 1$ and $2\gamma + 1$, expressing your answer in a form with integer coefficients. [7]

6 Prove by induction that $\sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1)$. [8]

Section B (36 marks)

7 A curve has equation $y = \dfrac{5}{(x+2)(4-x)}$.

(i) Write down the value of y when $x = 0$. [1]

(ii) Write down the equations of the three asymptotes. [3]

(iii) Sketch the curve. [3]

(iv) Find the values of x for which $\dfrac{5}{(x+2)(4-x)} = 1$ and hence solve the inequality

$$\frac{5}{(x+2)(4-x)} < 1.$$ [5]

8 It is given that $m = -4 + 2\mathrm{j}$.

(i) Express $\dfrac{1}{m}$ in the form $a + b\mathrm{j}$. [2]

(ii) Express m in modulus-argument form. [4]

(iii) Represent the following loci on separate Argand diagrams.

(*A*) $\arg(z - m) = \dfrac{\pi}{4}$ [2]

(*B*) $0 < \arg(z - m) < \dfrac{\pi}{4}$ [3]

9 Matrices **M** and **N** are given by $\mathbf{M} = \begin{pmatrix} 3 & 2 \\ 0 & 1 \end{pmatrix}$ and $\mathbf{N} = \begin{pmatrix} 1 & -3 \\ 1 & 4 \end{pmatrix}$.

(i) Find $\mathbf{M}^{-1}$ and $\mathbf{N}^{-1}$. [3]

(ii) Find $\mathbf{MN}$ and $(\mathbf{MN})^{-1}$. Verify that $(\mathbf{MN})^{-1} = \mathbf{N}^{-1}\mathbf{M}^{-1}$. [6]

(iii) The result $(\mathbf{PQ})^{-1} = \mathbf{Q}^{-1}\mathbf{P}^{-1}$ is true for any two 2×2, non-singular matrices **P** and **Q**.

The first two lines of a proof of this general result are given below. Beginning with these two lines, complete the general proof.

$$(\mathbf{PQ})^{-1}\mathbf{PQ} = \mathbf{I}$$
$$\Rightarrow \quad (\mathbf{PQ})^{-1}\mathbf{PQQ}^{-1} = \mathbf{IQ}^{-1}$$ [4]